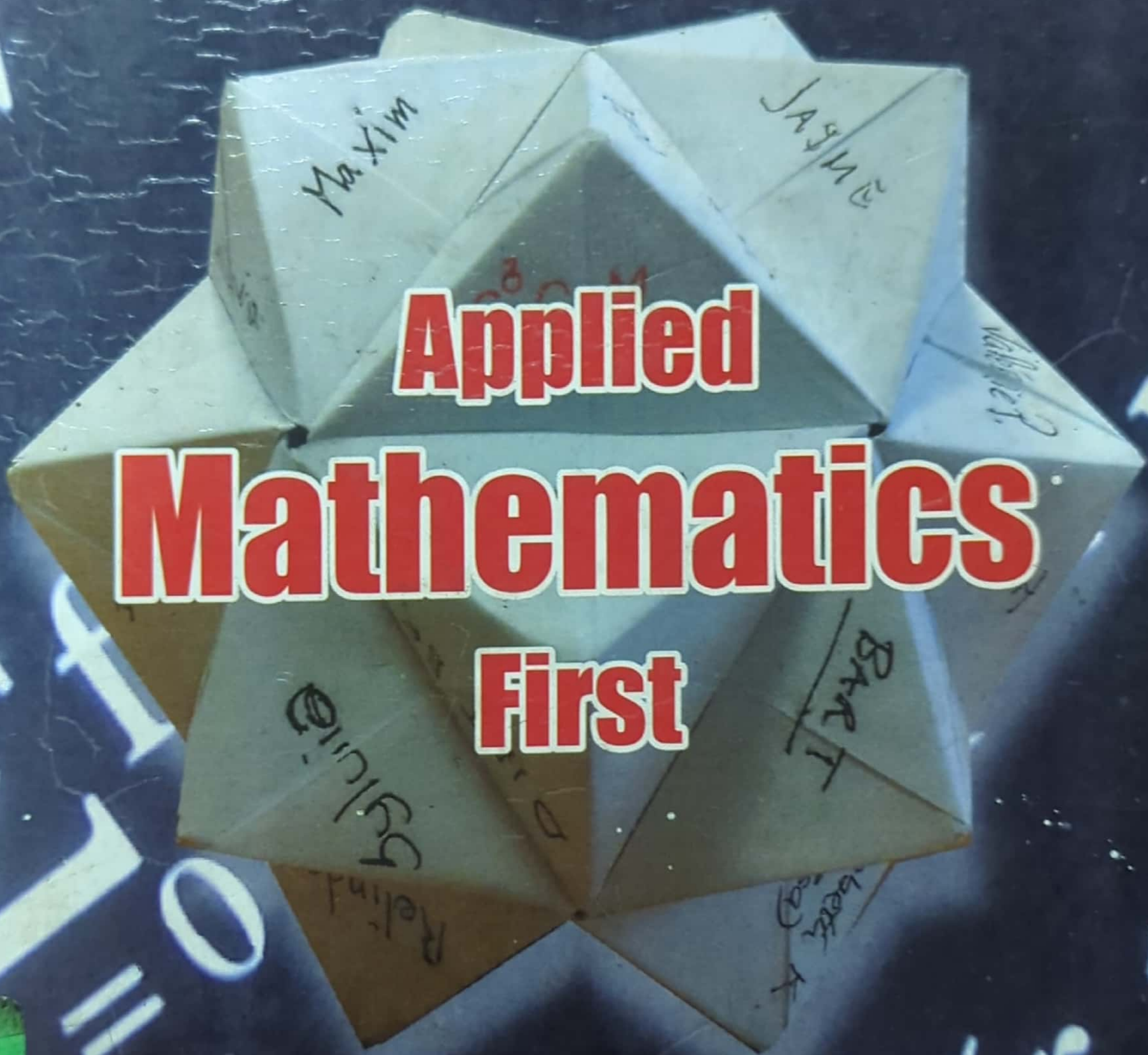
Deepak Prakashan



Applied Mathematics First

Vikas Shinde
Shashi S. Narwariya

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CHAPTER 1

DETERMINANTS

1.1 DETERMINANTS

Consider the two simultaneous homogeneous linear equations :

$$a_{11}x + a_{12}y = 0 \quad ...(i)$$

$$a_{21}x + a_{22}y = 0 \quad ...(ii)$$

From equation (i), we have

$$a_{11}x = -a_{12}y$$

or

$$\frac{x}{y} = -\frac{a_{12}}{a_{11}} \quad ...(iii)$$

Also, from equation (ii), we have

$$a_{21}x = -a_{22}y$$

or

$$\frac{x}{y} = -\frac{a_{22}}{a_{21}} \quad ...(iv)$$

Hence from equations (iii) and (iv), we get

$$-\frac{a_{12}}{a_{11}} = -\frac{a_{22}}{a_{21}}$$

$$\Rightarrow \quad a_{11}\,a_{22} - a_{12}\,a_{21} = 0$$

This result can also be written as

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = 0$$

Hence, it is clear that

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12} \quad ...(v)$$

The expression on the L.H.S. of equation (v) is called a determinant, which has a definite numerical value.

The letters $a_{11}, a_{12}, a_{21}, a_{22}$ are called the constituents or elements of the determinant.

(i) Rows : The series of elements in the horizontal lines from left to right in a determinant are called rows of the determinant. Different rows of the determinant are denoted by $R_1, R_2, R_3, \ldots$

In above determinant a_{11}, a_{12} are the elements of the first row R_1 where as a_{21}, a_{22} are the elements of the second row R_2.

(ii) Columns : The series of elements in the vertical lines from top to bottom in a determinant are called columns of the determinant. Different columns of the determinant are denoted by $C_1, C_2, C_3 \ldots$

In above determinant a_{11}, a_{21} are the elements of the first column C_1 where as a_{12}, a_{22} are the elements of the second column C_2.

(iii) Order of the Determinant : A determinant having n rows and n columns is called "determinant of n^{th} order".

$$\text{Thus,} \quad \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

an example of "third order determinant."

(iv) Notation : Generally a determinant is denoted by Δ or D.

(v) Constituents of Principal or Leading Diagonal : The series of quantities (or elements) in the diagonal line from left hand top corner to right hand bottom corner are called the elements or constituents of the principal diagonal, or leading diagonal.

In the above third order determinant a_{11}, a_{22}, a_{33} are the elements of the principal diagonal.

1.2 EXPANSION OF DETERMINANT OF SECOND ORDER

Consider a determinant of second order,

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

Its value is $a_{11}a_{22} - a_{21}a_{12}$

where a_{11}, a_{22} are the elements of principal diagonal.

$\therefore$ Value of the second order determinant.

= Product of the elements of principal diagonal

− Product of the remaining elements

1.3 DETERMINANT OF THIRD ORDER

Consider the three simultaneous homogeneous linear equations,

$$a_1x + b_1y + c_1z = 0 \qquad \ldots(i)$$

$$a_2x + b_2y + c_2z = 0 \qquad \ldots(ii)$$

$$a_3x + b_3y + c_3z = 0 \qquad \ldots(iii)$$

From equations (ii) and (iii) by cross multiplication,

$$\frac{x}{b_2c_3 - b_3c_2} = \frac{y}{a_3c_2 - a_2c_3} = \frac{z}{a_2b_3 - a_3b_2} = k \text{ (let)}$$

$$\therefore \quad \left.\begin{aligned} x &= k(b_2c_3 - b_3c_2) \\ y &= k(a_3c_2 - a_2c_3) \\ z &= k(a_2b_3 - a_3b_2) \end{aligned}\right\} \qquad \ldots(iv)$$

where, $k \neq 0$ (because if $k = 0$, then $x = 0, y = 0, z = 0$). Putting the values of x, y and z from equation (iv) in (i), we get

$$ka_1(b_2c_3 - b_3c_2) + kb_1(a_3c_2 - a_2c_3) + kc_1(a_2b_3 - a_3b_2) = 0$$

or $$a_1(b_2c_3 - b_3c_2) + b_1(a_3c_2 - a_2c_3) + c_1(a_2b_3 - a_3b_2) = 0$$

or $$a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2) = 0$$

or $$a_1\begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - b_1\begin{vmatrix} a_2 & a_3 \\ c_2 & c_3 \end{vmatrix} + c_1\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} = 0$$

This result can also be written as

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0 \qquad \ldots(v)$$

The expression on the L.H.S. of equation (v) is called the determinant of third order.

Note : In determinant the number of rows and columns is always equal.

1.4 MINORS AND COFACTORS

1.4.1 Minors : The minor of any constituent (or element) is the determinant obtained by omitting from the original determinant, the row and the column to which it belongs.

$$\text{Let} \qquad \Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

be a determinant of second order. Then, minor of the constituent a_{11} is

$$M_{11} = a_{22}$$

Minor of the constituent a_{12} is

$$M_{12} = a_{21}$$

Similarly, the expansion of the determinant with respect to the other rows or columns is possible as :

$$\Delta = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23}$$

[with respect to second row]

and

$$\Delta = a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33}$$

[with respect to third row]

etc.

ILLUSTRATIVE EXAMPLES

Example 1.1. *Find all the cofactors of the determinant* $\begin{vmatrix} 1 & 2 \\ -1 & -2 \end{vmatrix}$.

Sol. Let $\Delta = \begin{vmatrix} 1 & 2 \\ -1 & -2 \end{vmatrix}$

Cofactors of the element 1 is

$$A_{11} = (-1)^{1+1} M_{11} = M_{11} = -2$$

Cofactors of the element 2 is

$$A_{12} = (-1)^{1+2} M_{12} = -M_{12} = -(-1) = 1$$

Cofactors of the element – 1 is

$$A_{21} = (-1)^{2+1} M_{21} = -M_{21} = -2$$

Cofactors of the element – 2 is

$$A_{22} = (-1)^{2+2} M_{22} = M_{22} = 1$$

Hence, – 2, 1, – 2, 1 **Ans.**

Example 1.2. *Write all the cofactors of* $\begin{vmatrix} 1 & 2 & 3 \\ 4 & -5 & 6 \\ 7 & 0 & -1 \end{vmatrix}$.

Sol. Let $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 4 & -5 & 6 \\ 7 & 0 & -1 \end{vmatrix}$

$$A_{11} = (-1)^{1+1} M_{11} = \begin{vmatrix} -5 & 6 \\ 0 & -1 \end{vmatrix} = 5 - 0 = 5$$

$$A_{12} = (-1)^{1+2} M_{12} = -\begin{vmatrix} 4 & 6 \\ 7 & -1 \end{vmatrix} = -(-4 - 42) = 46$$

$$A_{13} = (-1)^{1+3} M_{13} = \begin{vmatrix} 4 & -5 \\ 7 & 0 \end{vmatrix} = 0 + 35 = 35$$

$$A_{21} = (-1)^{2+1} M_{21} = -\begin{vmatrix} 2 & 3 \\ 0 & -1 \end{vmatrix} = -(-2 - 0) = 2$$

$$A_{22} = (-1)^{2+2} M_{22} = \begin{vmatrix} 1 & 3 \\ 7 & -1 \end{vmatrix} = -1 - 21 = -22$$

$$A_{23} = (-1)^{2+3} M_{23} = -\begin{vmatrix} 1 & 2 \\ 7 & 0 \end{vmatrix} = -(0 - 14) = 14$$

$$A_{31} = (-1)^{3+1} M_{31} = \begin{vmatrix} 2 & 3 \\ -5 & 6 \end{vmatrix} = 12 + 15 = 27$$

$$A_{32} = (-1)^{3+2} M_{32} = -\begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} = -(6 - 12) = 6$$

$$A_{33} = (-1)^{3+3} M_{33} = \begin{vmatrix} 1 & 2 \\ 4 & -5 \end{vmatrix} = -5 - 8 = -13$$

Now, 5, 46, 35, 2, – 22, 14, 27, 6, – 13 **Ans.**

Example 1.3. *In the determinant* $\begin{vmatrix} 3 & -3 & 4 \\ 3 & 2 & -2 \\ -1 & 1 & 1 \end{vmatrix}$ *find the cofactors of the elements 3, 3, – 1 and hence find the value of the determinant.*

$= 2 : -2$

$= 1 : -1$

Ans.

EXERCISE 1.1

1. Find the values of all the cofactors of the following determinants :

(i) $\begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix}$ (ii) $\begin{vmatrix} 2 & -3 \\ 1 & -2 \end{vmatrix}$

(iii) $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$ (iv) $\begin{vmatrix} 1 & -1 & 1 \\ 1 & 2 & 3 \\ -1 & -3 & 2 \end{vmatrix}$

2. Find the cofactors of the elements of second column of the determinant

$$\begin{vmatrix} 1 & 2 & 3 \\ -4 & 3 & 6 \\ 2 & 7 & 9 \end{vmatrix}$$

3. Find the ratio of the cofactor and minor of 4 in the determinant

$$\begin{vmatrix} 0 & 1 & -8 \\ -1 & 0 & 4 \\ 2 & -1 & 0 \end{vmatrix}$$

4. In the determinant $\begin{vmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{vmatrix}$, find the cofactors of the elements 1, – 3, 2 and hence find the value of the determinant.

Answers

1. (i) 1, – 2, 0, 1
   (ii) – 2, – 1, 3, 2
   (iii) – 3, 6, – 3, 6, – 12, 6, – 3, 6, – 3
   (iv) 13, – 5, – 1, – 1, 3, 4, – 5, – 2, 3
2. 3, 3, – 3
3. 1 : – 1
4. – 12, – 2, 23 and 40

1.7 PROPERTIES OF DETERMINANTS

(i) If each constituent in any row, or in any column is zero, then the value of the determinant is zero.

Let $\Delta = \begin{vmatrix} 0 & 0 & 0 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

Then $\Delta = 0(b_2c_3 - b_3c_2) - 0(a_2c_3 - a_3c_2) + 0(a_2b_3 - a_3b_2)$

(ii) If rows be changed into columns and columns into rows, the value of the determinant remains unaltered.

Let $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

Then $\Delta = a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)$ ...(i)

Now let the determinant obtained by changing the columns into rows and rows into columns be Δ', then

$$\Delta' = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

or $\Delta' = a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)$

$= a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)$ ...(ii)

. Then

$\Delta' = \Delta.$

(iii) If two adjacent rows or columns of a determinant are interchanged, the sign of the value of determinant is changed.

Let $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

Then $\Delta = a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)$ ...(i)

Now let the determinant obtained by interchanging the first and second rows be Δ'. Then

$$\Delta' = \begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$= a_2(b_1c_3 - b_3c_1) - b_2(a_1c_3 - a_3c_1) + c_2(a_1b_3 - a_3b_1)$

$= -[a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)]$ ...(ii)

From equations (i) and (ii), we see that

$\Delta' = -\Delta$

Note : (i) The sign of a determinant will, or will not change according as there will be an odd or even number of interchanges, on the whole, among its columns or rows.

(ii) If a row or column of a determinant Δ is passed over n parallel rows or by $R_1 \to R_1 - R_3$

$$= \begin{vmatrix} 0 & 2 & 0 \\ 7 & 13 & 5 \\ 9 & 23 & 6 \end{vmatrix}$$

$$= -2 \begin{vmatrix} 7 & 5 \\ 9 & 6 \end{vmatrix}$$

$$= -2(7 \times 6 - 9 \times 5)$$

$$= 6$$ **Ans.**

Example 1.8. *Prove that :*

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = abc + 2fgh - af^2 - bg^2 - ch^2$$

Sol. Let $\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$

$$\Delta = a \begin{vmatrix} b & f \\ f & c \end{vmatrix} - h \begin{vmatrix} h & f \\ g & c \end{vmatrix} + g \begin{vmatrix} h & b \\ g & f \end{vmatrix}$$

$$= a(bc - f^2) - h(hc - fg) + g(hf - bg)$$

$$= abc - af^2 - ch^2 + fgh + fgh - bg^2$$

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$$ **Proved.**

Example 1.9. *Prove that :*

$$\begin{vmatrix} a^2 & bc & ac + c^2 \\ a^2 + ab & b^2 & ac \\ ab & b^2 + bc & c^2 \end{vmatrix} = 4a^2b^2c^2$$

Sol. Let $\Delta = \begin{vmatrix} a^2 & bc & ac + c^2 \\ a^2 + ab & b^2 & ac \\ ab & b^2 + bc & c^2 \end{vmatrix}$

$$= abc \begin{vmatrix} a & c & a + c \\ a + b & b & a \\ b & b + c & c \end{vmatrix}$$

by $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1 - C_2$

$$= abc \begin{vmatrix} a & c - a & 0 \\ a + b & -a & -2b \\ b & c & -2b \end{vmatrix}$$

$$= abc(-2b) \begin{vmatrix} a & c - a & 0 \\ a + b & -a & 1 \\ b & c & 1 \end{vmatrix}$$

by $R_2 \to R_2 - R_3$

$$= abc(-2b) \begin{vmatrix} a & c - a & 0 \\ a & -a - c & 0 \\ b & c & 1 \end{vmatrix}$$

Expanding with respect to C_3

$$= -2b(abc).1 \begin{vmatrix} a & c - a \\ a & -a - c \end{vmatrix}$$

$$= -2b(abc)[a(-a - c) - a(c - a)]$$

$$= -2ab(abc)[-a - c - c + a]$$

$$= -2ab(abc)(-2c)$$

$$\Delta = 4a^2b^2c^2$$ **Proved.**

Example 1.10. *Prove that :*

$$\begin{vmatrix} \frac{a^2 + b^2}{c} & c & c \\ a & \frac{b^2 + c^2}{a} & a \\ b & b & \frac{c^2 + a^2}{b} \end{vmatrix} = 4abc$$

Sol. Let $\Delta = \begin{vmatrix} \frac{a^2 + b^2}{c} & c & c \\ a & \frac{b^2 + c^2}{a} & a \\ b & b & \frac{c^2 + a^2}{b} \end{vmatrix}$

Taking common $\frac{1}{c}$ from R_1, $\frac{1}{a}$ from R_2 and $\frac{1}{b}$ from R_3

$$= \frac{1}{abc} \begin{vmatrix} a^2 + b^2 & c^2 & c^2 \\ a^2 & b^2 + c^2 & a^2 \\ b^2 & b^2 & c^2 + a^2 \end{vmatrix}$$

or $(p-a)(q-b)r + (p-a)(r-c)b + (q-b)(r-c)a = 0$

Dividing by $(p-a)(q-b)(r-c)$ we have

$$\frac{r}{r-c} + \frac{b}{q-b} + \frac{a}{p-a} = 0$$

or $$\frac{r}{r-c} + \frac{b-q+q}{q-b} + \frac{a-p+p}{p-a} = 0$$

or $$\frac{r}{r-c} + \frac{q}{q-b} - 1 + \frac{p}{p-a} - 1 = 0$$

$\therefore$ $$\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$$ **Proved.**

EXERCISE 1.2

1. Evaluate the following determinants :

(i) $\begin{vmatrix} 12 & 3 & 7 \\ 27 & 7 & 17 \\ 36 & 9 & 22 \end{vmatrix}$ (ii) $\begin{vmatrix} 265 & 240 & 219 \\ 240 & 225 & 198 \\ 219 & 198 & 225 \end{vmatrix}$

(iii) $\begin{vmatrix} 13 & 3 & 23 \\ 30 & 7 & 53 \\ 39 & 9 & 70 \end{vmatrix}$ (iv) $\begin{vmatrix} 21 & 17 & 7 & 10 \\ 24 & 22 & 6 & 10 \\ 6 & 8 & 2 & 3 \\ 5 & 7 & 1 & 2 \end{vmatrix}$

(v) $\begin{vmatrix} 2^2 & 3^2 & 4^2 & 5^2 \\ 3^2 & 4^2 & 5^2 & 6^2 \\ 4^2 & 5^2 & 6^2 & 7^2 \\ 5^2 & 6^2 & 7^2 & 8^2 \end{vmatrix}$ (vi) $\begin{vmatrix} a^3 & a^2 & a & 1 \\ b^3 & b^2 & b & 1 \\ c^3 & c^2 & c & 1 \\ d^3 & d^2 & d & 1 \end{vmatrix}$

(vii) $\begin{vmatrix} 1 & x & x^2-yz \\ 1 & y & y^2-zx \\ 1 & z & z^2-xy \end{vmatrix}$ (viii) $\begin{vmatrix} 265 & 240 & 219 \\ 240 & 225 & 198 \\ 219 & 198 & 181 \end{vmatrix}$

2. Prove the following :

(i) $\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 2(3abc - a^3 - b^3 - c^3)$

(ii) $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

(iii) $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

(iv) $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left[\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + 1\right]$

(v) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

(vi) $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

(vii) $\begin{vmatrix} a+b & c & c \\ a & b+c & a \\ b & b & c+a \end{vmatrix} = 4abc$

(viii) $\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix} = (b^2-ac)(ax^2+2bxy+cy^2)$

(ix) $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ zy & zx & xy \end{vmatrix} = (y-z)(z-x)(x-y)(yz+zx+xy)$

(x) $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$

(xi) $\begin{vmatrix} y+z & x & y \\ z+x & z & x \\ x+y & y & z \end{vmatrix} = (x+y+z)(x-z)^2$

(xii) $\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$

(xiii) $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$

(xiv) $\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} = \begin{vmatrix} y & b & p \\ x & a & q \\ z & c & r \end{vmatrix} = \begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix}$

2 MEAN AND ROOT MEAN SQUARE VALUE

CHAPTER

2.1 MEAN VALUE

It is often necessary to know the mean value of a varying quantity. For example the mean value of the power generated over a certain period is the mean value of the products of current and electromotive force measured at various instants during that period.

The mean value of a series of values is found by adding the values together and dividing the sum by the number of values taken i.e., if the values are $x_1, x_2, x_3, ..., x_n$ and number of values obviously are '*n*' then the mean,

$$M = \frac{x_1 + x_2 + x_3 \text{ .... } + x_n}{n}$$

$$= \frac{\Sigma x_i}{n}$$

If the value x_1 occurs f_1 times, the value x_2 occurs f_2 times, ..., the value x_n occurs f_n times, then the mean value

$$M = \frac{f_1x_1 + f_2x_2 + ... + f_nx_n}{N} = \frac{\Sigma f_i x_i}{N}$$

where, $N = f_1 + f_2 + ... + f_n = \Sigma f_i$ (total frequency)

ILLUSTRATIVE EXAMPLES

Example 2.1. *The values of the current (in amperes) for a full period in a certain circuit are as follows :*

0, – 1·8, – 3·5, – 5·0, – 6·1, – 6·8, – 6·1, – 5·0, – 3·5, – 1·8, 0, 1·8, 3·5, 5·0, 6·1, 6·8, 6·1, 5·0, 3·5, 1·8, 0.

Find the mean value of the current during the period these values were recorded.

Sol. Obviously here sum of the values = 0

Number of values = 21

Therefore mean value of the current $= \frac{0}{21}$

= 0 amperes Ans.

Example 2.2. *The values of electric current 'C' and electromotive force 'E' in a certain circuit are as follows :*

C (amperes)	0	1·6	2·8	3·5	4·0	5·2	4·0	3·5	2·8	1·6
E (volts)	0	10·2	14·6	19·6	25·4	32·8	25·4	19·6	14·6	10·2

Find the mean value of the power over the period during which these values were taken.

Sol. We know that

Power = electric current × electromotive force

Now, mean value of the power

$$= \frac{\Sigma C_i E_i}{n}, \text{ where } n = 10$$

Electric Current (C_i)	*Electromotive Force* (E_i)	$C_i E_i$
0	0	0
1·6	10·2	16·32
2·8	14·6	40·88
3·5	19·6	68·6
4·0	25·4	101·6
5·2	32·8	170·56
4·0	25·4	101·6
3·5	19·6	68·6
2·8	14·6	40·88
1·6	10·2	16·32
$n = 10$		$\Sigma C_i E_i = 625·36$

$$\text{Mean value of the power} = \frac{625·36}{10}$$

= 62·536 watts Ans.

Example 2.3. Find the mean for the following data :

Height (in cm)	219	216	213	210	207	204	201	198	195
No. of Persons	2	4	6	10	11	7	5	4	1

Sol.

Height (x_i)	*No. of Persons* (f_i)	$f_i x_i$
219	2	438
216	4	864
213	6	1278
210	10	2100
207	11	2277
204	7	1428
201	5	1005
198	4	792
195	1	195
	$\Sigma f_i = 50$	$\Sigma f_i x_i = 10377$

Now, $\text{Mean } M = \dfrac{\ldots}{\Sigma f_i}$

$$= \frac{10377}{50}$$

$$= \mathbf{207{\cdot}54}$$

Example 2.4. *Following are the weights (in kg) of 10 persons 72, 76, 70, 62, 50, 71, 90, 64, 58, 82 find the mean value.*

Sol. We know that

$$\text{Mean value} = \frac{\Sigma x_i}{n}$$

$$\Sigma x_i = 72 + 76 + 70 + 62 + 50 + 71 + 90 + 64 + 58 + \ldots$$

$$= 695$$

$$n = 10$$

$$\text{Mean value} = \frac{695}{10}$$

$$= \mathbf{69{\cdot}5\ kg}$$

Ans.

EXERCISE 2.1

1. Find the mean value of the values 5, 15, 12, 18, 10.
2. The mean value of 10 values is 50, then find the sum of these values.
3. The values of electric current C and electromotive force (e.m.f.) E in a certain circuit are as follows :

   C (ampere) : 0, 1·02, 1·92, 2·12, 2·58, 2·84, 3·0, 2·84, 2·58, 2·12, 1·92, 1·02, 0

   E (volt) : 0, 3·42, 6·42, 7·07, 8·66, 9·84, 10·0, 9·84, 8·66, 7·07, 6·42, 3·42, 0

   Find the mean value of power.
4. In a certain mine the set up of the staff is as follows :

Personnel	*Salary Rs. per month*	*Number*
Manager	2400	1
Under Manager	860	3
Foreman	440	15
Mate	220	40
Labour	120	250

Calculate the average salary per person and average expenditure on salary per day (1 month = 30 days).

5. In a certain state the rain in a year of 12 districts is given by x_i (in inches) along with the number of rain measuring centre (w_i) is given as follows :

x_i :	15	16	18	12	10	14	16	15	21	8	9	17
w_i :	10	6	5	8	25	7	18	15	10	30	20	21

Find the average rain for this year.

6. Calculate the mean value for the following data :

x :	5	10	15	20	25	30	35	40
f :	35	80	135	220	150	330	105	80

Answers

1. 12 **2.** 500

3. 14·78 watt **4.** Rs. 163·04, Rs. 1679·33

5. 19·91 **6.** 6·385 m

2.2 ROOT MEAN SQUARE VALUE

In example 2.1, we have seen that we get a zero mean for an alternating current. The mean thus fails to give an idea of the variation in magnitude of the current in case of the sum of positive terms and sum of negative terms are numerically equal. To get a measure of this variation we calculate the square root of the mean of the squares of the given values.

Since the square of a number, whether negative or positive is always positive. We do not get zero or a negative value for the average. The square root of the mean of the squares is called the root mean square value written in short as R.M.S. value. It is also known as effective value in the case of alternating current and alternating electromotive force.

If the series is $x_1, x_2, x_3 \ldots x_n$ then its R.M.S. value

$$= \sqrt{\frac{x_1^2 + x_2^2 + \ldots + x_n^2}{n}}$$

ILLUSTRATIVE EXAMPLES

Example 2.5. *Find the root mean square value of the values 3, 13, 23, – 33, – 43.*

Sol. Given values are

3, 13, 23, – 33, – 43

The square of the given values are

9, 169, 529, 1089, 1849

Sum of the squares = 3645

CHAPTER 3

QUADRATIC EQUATIO

3.1 POLYNOMIAL

We are already familiar with the algebraic expression li $5x^2+7x+8$, $5x^4+3x^2+3$ etc. These expression are called po mials. A polynomial involving only one variable is called the po mial in one variable.

3.1.1 Quadratic Polynomial : A polynomial of degree 2 in one v able is called quadratic polynomial. The most general form of a ax^2+bx+c, where $a, b, c, \in R, a \neq 0$.

3.2 GENERAL QUADRATIC EQUATION

An equation in one unknown, in which the highest power of unknown is two, is called a quadratic equation or simply an equatio the second degree. Thus the equation $ax^2+bx+c=0$, in which $a \neq 0$ that the highest power of unknown x is two, is called a quadratic eq tion. It means that a quadratic equation must contain at least one te involving the square of unknown.

3.2.1 Types of Quadratic Equations : Quadratic equations ar two types :

(i) **Pure Quadratic Equation :** A quadratic equation which first degree term of variable (unknown) is missing is called pu quadratic equation. For example, $ax^2+c=0$, $x^2+5=0$ etc.

(ii) **Affected or Complete Quadratic Equation :** A qua ratic equation which contains both first and second degree terms variable is called affected or complete quadratic equation. For e ample, $ax^2+bx+c=0,\ a \neq 0,\ x^2-5x+5=0$ etc.

3.3 SOLUTION OF PURE QUADRATIC EQUATION

$$ax^2+b = 0, a \neq 0$$

$$\Rightarrow \quad ax^2 = -b \qquad \text{(Transposing 'b' to R.H.S.)}$$

$$\Rightarrow \quad x^2 = -b/a$$

$$\Rightarrow \quad x = \pm\sqrt{-b/a}$$

Here, $\sqrt{-b/a}$ will be an imaginary number if $a > 0$ and $b > 0$.

3.4 SOLUTION OF AFFECTED QUADRATIC EQUATION

Method I. Solution by Factorization : If the left hand member of the standard form of the quadratic can be readily factorized into two linear factors, this ensures solution of the equation.

For example to solve $x^2=6x-8$, we put in the standard form i.e.,

$$x^2-6x+8 = 0$$

$$\Rightarrow \quad x^2-4x-2x+8 = 0$$

$$\Rightarrow \quad x(x-4)-2(x-4) = 0$$

$$\Rightarrow \quad (x-4)(x-2) = 0$$

Therefore either $x-4 = 0 \Rightarrow x=4$

or $x-2 = 0 \Rightarrow x=2$

or $x = 4, 2$

Two solutions are thus obtained.

Method II. Solution by Formula Method : This is general method of solving a quadratic. Let the quadratic be $ax^2+bx+c=0$, where $a \neq 0$, we can write the equation as

$$x^2+\frac{b}{a}x+\frac{c}{a} = 0$$

$$\Rightarrow \quad x^2+\frac{b}{a}x = -\frac{c}{a}$$

[adding $\left(\frac{b}{2a}\right)^2$ both sides to make perfect square]

$$\Rightarrow \quad x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2-\frac{c}{a}$$

$$\Rightarrow \quad \left(x+\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}-\frac{c}{a}$$

$$\Rightarrow \quad \left(x+\frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$$

$$\Rightarrow \quad x+\frac{b}{2a} = \frac{\pm\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow \quad x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Hence two roots of the equation are

$$\frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \frac{-b-\sqrt{b^2-4ac}}{2a}$$

Now $a, b, c \in R \Rightarrow b^2 - 4ac \in R$

We recall $b^2 - 4ac$, discriminant of the equation and gen... denoted by D. We discriminate the roots of the equation with the... of D.

3.5 ROOTS OF QUADRATIC EQUATION

The values of the variable involved in the equation which sa...fy the equation are called the roots of the given equation. For exam...

$$\begin{aligned} \text{If} \quad x^2 - 9 &= 0 \\ \Rightarrow \quad x^2 &= 9 \\ \Rightarrow \quad x &= \pm 3 \end{aligned}$$

Hence 3, – 3 both are the roots of the equation $x^2 - 9 = 0$ be...cause both satisfy the equation, as

If $\quad x = 3, x^2 - 9 = (3)^2 - 9 = 9 - 9 = 0 = \text{R.H.S.}$

and $\quad x = -3, x^2 - 9 = (-3)^2 - 9 = 9 - 9 = 0 = \text{R.H.S.}$

Note : Sometimes we face the situation that all the values of variable after sol...ing an equation do not satisfy the equation such a values of variable is called "extraneou... root."

3.5.1 A General Quadratic Equation can not have More Than Two Distinct Roots

Proof : If possible let α, β, γ be three different roots of the equation $ax^2 + bx + c = 0, a \neq 0$.

Since α, β, γ are the roots we have

$$a\alpha^2 + b\alpha + c = 0 \quad \text{...(i)}$$

$$a\beta^2 + b\beta + c = 0 \quad \text{...(ii)}$$

$$a\gamma^2 + b\gamma + c = 0 \quad \text{...(iii)}$$

Subtracting equation (ii) from (i), we have

$$a(\alpha^2 - \beta^2) + b(\alpha - \beta) = 0$$

or $\quad (\alpha - \beta)\,|a(\alpha + \beta) + b| = 0$

Since α is different from β so $\alpha - \beta$ can not be equal to zero

$\Rightarrow \quad a(\alpha + \beta) + b = 0 \quad \text{...(iv)}$

Similarly from equations (ii) and (iii), we have

$$a(\beta + \gamma) + b = 0 \quad \text{...(v)}$$

Now from equations (iv) and (v), we have

$$a[(\alpha + \beta) - (\beta + \gamma)] = 0$$

$$\Rightarrow \quad a(\alpha - \gamma) = 0 \quad \text{...(vi)}$$

It is impossible since by hypothesis $\alpha \neq \gamma$ as well as $a \neq 0$

Hence there can not be three different roots.

Note : If α, β, γ be three different values of x satisfying $ax^2 + bx + c = 0$. Then it follows from equation (vi), that $a = 0$, and hence from equation (v) and equation (i) $b = 0, c = 0$. Thus the equation reduces to the form $0.x^2 + 0.x + 0 = 0$, which is evidently satisfied by any value of x and is therefore an identity. Thus, a quadratic equation is an identity if it is satisfied by more than two unequal values of the unknown.

ILLUSTRATIVE EXAMPLES

Example 3.1. *Solve the quadratic equation $2x^2 + 3x - 2 = 0$.*

Sol. **First Method :** On factorizing the given equation

$$\begin{aligned} 2x^2 + 3x - 2 &= 0 \\ \Rightarrow \quad 2x^2 + 4x - x - 2 &= 0 \\ \Rightarrow \quad 2x(x+2) - 1(x+2) &= 0 \\ \Rightarrow \quad (2x-1)(x+2) &= 0 \\ \therefore \quad 2x - 1 &= 0 \Rightarrow x = \frac{1}{2} \\ \text{or} \quad x + 2 &= 0 \Rightarrow x = -2 \\ x &= \frac{1}{2}, -2 \end{aligned}$$

Ans.

Second Method : By formula method, we have on comparing the given equation by general equation $ax^2 + bx + c = 0$.

Here, $a = 2, b = 3, c = -2$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-3 \pm \sqrt{(3)^2 - 4 \times 2 \times (-2)}}{2 \times 2} \\ &= \frac{-3 \pm \sqrt{25}}{4} \\ x &= \frac{-3 \pm 5}{4} \end{aligned}$$

Taking '+' sign, $\quad x = \frac{-3+5}{4} = \frac{1}{2}$

OBJECTIVE QUESTIONS

1. If $f(x) = \sqrt{\frac{x+1}{3x+1}}$ then $f\left\{\sin^2\left(\frac{\pi}{4}\right)\right\}$ is equal to :

(A) $\sqrt{\frac{3}{5}}$ (B) $\frac{3}{5}$

(C) $\sqrt{\frac{2}{3}}$ (D) $\frac{2}{3}$

2. The function $f(x) = ax^2 + b$ is :

(A) constant function (B) even function

(C) even function (D) none of these

3. If $\phi(\theta) = \sin^2\theta$ then $\phi\left(\frac{\pi}{4}\right)$ is equal to :

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$

(C) $\frac{1}{4}$ (D) 1

4. If $f(x) = 3\sin x + \tan^2 x$ then $f\left(\frac{\pi}{3}\right)$ is equal to :

(A) $\frac{3}{2}(\sqrt{3}+2)$ (B) $3(\sqrt{3}+2)$

(C) $\sqrt{3}(3+\sqrt{2})$ (D) $3(\sqrt{3}+\sqrt{2})$

5. If $f(x) = \frac{3x^2+5x+3}{x^2+1}$ then :

(A) $f(x) = f(-x)$ (B) $f(x) = 2f(-x)$

(C) $f(x) = f\left(\frac{1}{x}\right)$ (D) $f(x) = f\left(-\frac{1}{x}\right)$

Objective Answersheet

Q.	1	2	3	4	5
Ans.	(A)	(C)	(B)	(A)	(C)

13 CHAPTER LIMIT

13.1 CONCEPT OF LIMIT

Suppose we have 1 kg of rice. If we throw away half of the rice, we are left with 500 gms i.e., $\frac{1}{2}$ kg. If we again throw away half of rice of what is left, we are left with $\frac{1}{4}$ kg, i.e., $\frac{1}{2^2}$ kg. Again if we throw half of remaining rice we shall have $\frac{1}{2^3}$ kg of rice.

In this way if half portion of rice is thrown every time then the remainder after repeating the processes n time is $\frac{1}{2^n}$ kg. If the process of throwing half the remainder is performed a large number of times, the rice left with us in the end is very little. Thus we say if n is very large then $\frac{1}{2^n}$ is very, very small. Mathematically we say that as n tends to infinity ($n \to \infty$), $\frac{1}{2^n}$ tends to zero $\left(\frac{1}{2^n} \to 0\right)$, and it is symbolically written as follows :

$$\lim_{n \to \infty} \frac{1}{2^n} = 0, \text{ here 'lim' is used for limit.}$$

13.1.1 Definition of Limit : The limit of a function for an assigned value of the independent variable is that value from which the function may be made to differ by less than any assignable quantity however small by making the independent variable approach sufficiently near its assigned value.

Methematically we can say that a function $f(x)$ is said to tend to a limit l as $x \to a$ if for a given positive number $\epsilon > 0$, there exists a positive number $\delta > 0$ such that

$$|f(x) - l| < \epsilon \text{ for } 0 < |x - a| < \delta$$

Now that $f(x)$ will tend to l as x approaches 'a' x approaches 'a' does not mean $x = a$. We denote it as

$$\lim_{x \to a} f(x) = l$$

(ii) $\lim\limits_{x \to a} [f(x) \cdot g(x)] = \lim\limits_{x \to a} f(x) \cdot \lim\limits_{x \to a} g(x) = A \cdot B$

(iii) $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x \to a} f(x)}{\lim\limits_{x \to a} g(x)} = \dfrac{A}{B}$

(iv) $\lim\limits_{x \to a} [c\,f(x)] = c \cdot \lim\limits_{x \to a} f(x) = cA$

where c is a constant.

13.5 MEANING OF LEFT LIMIT AND RIGHT LIMIT

The definition of limit implies that $f(x)$ approaches the same limit l irrespective of the manner in which x approaches a. Whether from right or left, x may approach a from right, i.e., x may take values nearer and nearer to a, remaining always greater than a or x may approach from left, i.e., it may take values nearer and nearer to a, remaining always less than a.

If $f(x)$ tends to a limit l_1, when x tends to a from right (i.e., from values greater than a).

This limit of $f(x)$ is called right handed limit of $f(x)$ and is symbolically written as

$$\lim_{x \to a^+} f(x) = l_1$$

a^+ indicates that x approaches a through values greater than a.

If $f(x)$ tends to a limit l_2 when x tends to a from left (i.e., from values smaller than a) this limit of $f(x)$ is called left handed limit of $f(x)$ and is symbolically written as :

$$\lim_{x \to a^-} f(x) = l_2$$

a^- indicates that x approaches a through values smaller than a.

The statement $\lim\limits_{x \to a} f(x) = l$ states that both the right handed and the left handed limits exist and each is equal to l.

Thus for the existance of a unique limit of $f(x)$ as $x \to a$ the necessary and sufficient condition is

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

or $$f(a-0) = f(a+0)$$

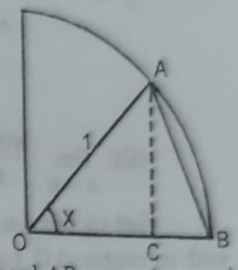
13.6 SOME IMPORTANT LIMITS

(i) $\lim\limits_{x \to 0} \sin x = 0$

Let us draw a circle of radius 1 with its centre O.

Radius, $OA = r = 1$

Let $\angle AOB = \theta = x < \dfrac{\pi}{2}$



In $\Delta\, OCA$,

$$AC = OA \sin x = \sin x$$

$\because$ $$AC < \text{chord } AB$$

$\therefore$ $$\sin x < x \qquad [\because \text{Chord } AB = r x = 1 \cdot x = x]$$

Now $0 < |\sin x| < |x|$ when $0 < |x| < \dfrac{\pi}{2}$

Let, h is a real number and $0 < h < \dfrac{1}{2}$

$\because \dfrac{1}{2} < \dfrac{\pi}{2} \Rightarrow |x| < \dfrac{1}{2} < h \Rightarrow |\sin x| < h$

$\therefore$ $|\sin x - 0| = |\sin x| < h$ when $|x - 0| = |x| < h$

$\Rightarrow |f(x)| < h \quad \therefore f(x) = \sin x$

By definition of limit

$$\sin x = 0 \text{ when } x \to 0$$

$$\lim_{x \to 0} \sin x = 0$$

Alternative Method : If x is smaller then expension of $\sin x$ is

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$$

$\therefore$ $$\lim_{x \to 0} \sin x = \lim_{x \to 0} \left[x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots\right] = 0$$

(ii) $\lim\limits_{x \to 0} \cos x = 1$

$$\lim_{x \to 0} \cos x = \lim_{x \to 0} \left(1 - 2\sin^2 \frac{x}{2}\right)$$

$$= \lim_{x \to 0} 1 - \lim_{x \to 0} 2\sin^2 \frac{x}{2}$$

$$= 1 - 2 \times 0 = 1$$

29. $\frac{m}{n} a^{m-n}$

30. $\frac{1}{2\sqrt{x}}$

31. e^2

34. 0

OBJECTIVE QUESTIONS

1. The value of $\lim_{x \to 2} \frac{\sin \pi x}{x}$ is :
   (A) π (B) $\pi/2$
   (C) $\frac{2\pi}{3}$ (D) 0

2. The value of $\lim_{x \to \alpha} \frac{\sin x - \sin \alpha}{x - \alpha}$ is :
   (A) $\cos \alpha$ (B) $\sin \alpha$
   (C) 0 (D) none of these

3. The value of $\lim_{n \to \infty} \frac{3n(4n+1)}{(n+4)(n+1)}$ is :
   (A) 4 (B) 8
   (C) 10 (D) 12

4. The value of $\lim_{x \to 2} \frac{x^2 - x - 2}{x - 2}$ is :
   (A) ∞ (B) 3
   (C) 2 (D) 1

5. The value of $\lim_{x \to 1} e^{-1/x}$ is :
   (A) 0 (B) ∞
   (C) does not exist (D) none of these

6. The value of $\lim_{n \to \infty} \frac{1^3 + 2^3 + 3^3 + \dots + n^3}{n^4}$ is :
   (A) 0 (B) ∞
   (C) 4 (D) 1/4

Objective Answersheet

Q.	1	2	3	4	5	6
Ans.	(D)	(A)	(D)	(B)	(C)	(D)

CHAPTER 14 DIFFERENTIATION BY FIRST PRINCIPLE

14.1 DEFINITION OF DERIVATIVE

Let $y = f(x)$ be a function of a real variable defined at any point $x_0 \in (a, b)$ and let the value of y be y_0 corresponding to x_0 i.e., $y_0 = f(x_0)$.

When x changes from x_0 to $x_0 + h$, the value of y_0 changes from y_0 to $y_0 + k$. If h is very small the k will also be very small, but the ratio $\frac{k}{h}$ will have a finite value (either small or large). If limit of the ratio $\frac{k}{h}$ as $h \to 0$ exists then this limiting value is called the derivative of y with respect to x at $x = x_0$.

The process of finding the differential coefficient is called differentiation.

We can explain differentiation as follows :

Let $\quad y_0 = f(x_0) \quad$ ...(i)

and $\quad y_0 + k = f(x_0 + h) \quad$ ...(ii)

From equations (i) and (ii), we get

$$k = f(x_0 + h) - f(x_0)$$

or $$\frac{k}{h} = \frac{f(x_0 + h) - f(x_0)}{h} \quad \text{[dividing by 'h']}$$

Taking the limit as $h \to 0$, we get

$$\lim_{h \to 0} \frac{k}{h} = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

Therefore the differential coefficient of $f(x)$ is given by

$$\lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h} \text{ at } x = x_0$$

Generally the differential coefficient of $f(x)$ at any point x is given by

$$\lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$$\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right) = \frac{-1}{x\,(2\sqrt{x})} = -\frac{1}{2x\sqrt{x}}$$

(iii) Differential Coefficient of $(ax+b)^n$.

Let $\quad y = f(x) = (ax+b)^n$

$$f(x+h) = [a(x+h)+b]^n.$$

By definition,

$$\frac{d}{dx}(ax+b)^n = \lim_{h\to 0}\frac{[a(x+h)+b]^n-(ax+b)^n}{h}$$

$$= \lim_{h\to 0}\frac{(ax+b)^n\left[1+\frac{ah}{(ax+b)}\right]^n-(ax+b)^n}{h}$$

$$= \lim_{h\to 0}\frac{(ax+b)^n\left[\left(1+\frac{ah}{ax+b}\right)^n-1\right]}{h}$$

$$= \lim_{h\to 0}\frac{(ax+b)^n}{h}\left[1+n\,.\frac{ah}{ax+b}+\frac{n(n-1)}{1.2}\left(\frac{ah}{ax+b}\right)^2+\ldots-1\right]$$

$$= \lim_{h\to 0}\frac{(ax+b)^n}{h}\left[n\,.\frac{ah}{(ax+b)}+\frac{n(n-1)}{1.2}\left(\frac{ah}{ax+b}\right)^2+\ldots\right]$$

$$= (ax+b)^n\left[\frac{na}{ax+b}+0\right]$$

$$\frac{d}{dx}(ax+b)^n = na\,(ax+b)^{n-1}$$

14.2.2 Differential Coefficient of Trigonometric Functions by First Principle

(i) Differential Coefficient of cos x.

Let $\quad f(x) = \cos x$

so that $f(x+h) = \cos(x+h)$

$$\therefore \quad \frac{d}{dx}f(x) = \lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h}$$

$$= \lim_{h\to 0}\frac{2\sin\left(x+\frac{h}{2}\right)\sin\left(-\frac{h}{2}\right)}{h}$$

$$= \lim_{h\to 0} -\sin\left(x+\frac{h}{2}\right)\times\lim_{h\to 0}\frac{\sin\frac{h}{2}}{\frac{h}{2}}$$

$$= (-\sin x)\times 1$$

Hence, $\quad \frac{d}{dx}(\cos x) = -\sin x$

(ii) Differential Coefficient of cot x.

Let $\quad y = \cot x = f(x)$

so that $f(x+h) = \cot(x+h)$

$$\therefore \quad \frac{dy}{dx} = \lim_{h\to 0}\frac{\cot(x+h)-\cot x}{h}$$

$$= \lim_{h\to 0}\frac{1}{h}\left[\frac{\cos(x+h)}{\sin(x+h)}-\frac{\cos x}{\sin x}\right]$$

$$= \lim_{h\to 0}\frac{1}{h}\cdot\frac{\sin x.\cos(x+h)-\sin(x+h).\cos x}{\sin x.\sin(x+h)}$$

$$= \lim_{h\to 0}\frac{1}{h}\cdot\frac{\sin(x-x-h)}{\sin(x+h).\sin x}$$

$$= \lim_{h\to 0} -\left(\frac{\sin h}{h}\right)\cdot\frac{1}{\sin(x+h).\sin x}$$

$$= \frac{-1}{\sin x.\sin x} = -\operatorname{cosec}^2 x$$

Hence, $\quad \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

(iii) Differential Coefficient of $\sin x^2$.

Let $\quad y = f(x) = \sin x^2$

$$f(x+h) = \sin(x+h)^2$$

By first principle

$$\frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\frac{d}{dx}(\sin x^2) = \lim_{h\to 0}\frac{\sin(x+h)^2-\sin x^2}{h}$$

$$\therefore \quad \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+\tan^2 y} = \frac{1}{1+x^2}$$

14.2.4 Differential Coefficient of Exponential Functions by First Principle

(i) Differential Coefficient of a^x.

Let $\quad y = a^x$

and $\quad y + k = a^{x+h}$

$\Rightarrow \quad k = a^{x+h} - y$

$\Rightarrow \quad k = a^{x+h} - a^x$

$$\Rightarrow \quad \frac{k}{h} = \frac{a^x . a^h - a^x}{h}$$

$$\therefore \quad \lim_{h\to 0} \frac{k}{h} = \lim_{h\to 0} \frac{a^x . a^h - a^x}{h}$$

$$\Rightarrow \quad \frac{dy}{dx} = \lim_{h\to 0} \frac{a^x (a^h - 1)}{h}$$

But $\quad a^h = 1 + h \log_e a + \frac{h^2}{2\,!}(\log_e a)^2 + \ldots$

$$\Rightarrow \quad \frac{dy}{dx} = \lim_{h\to 0} \frac{a^x \left[1 + h\log_e a + \frac{h^2}{2\,!}(\log_e a)^2 + \ldots - 1\right]}{h}$$

$$= \lim_{h\to 0} a^x \left(\log_e a + \frac{h}{2\,!}(\log_e a)^2 + \ldots\right)$$

$$= a^x \log_e a + 0 + \ldots$$

$$\therefore \quad \frac{d}{dx}(a^x) = a^x \log_e a$$

(ii) Differential Coefficient of e^x

Let $\quad y = e^x$

and $\quad y + k = e^{x+h}$

$\Rightarrow \quad k = e^{x+h} - y$

$\Rightarrow \quad k = e^{x+h} - e^x$

$$\Rightarrow \quad \frac{k}{h} = \frac{e^{x+h} - e^x}{h}$$

$$\therefore \quad \lim_{h\to 0} \frac{k}{h} = \lim_{h\to 0} \frac{e^x . e^h - e^x}{h}$$

$$\Rightarrow \quad \frac{dy}{dx} = \lim_{h\to 0} \frac{e^x (e^h - 1)}{h}$$

$$\Rightarrow \quad \frac{dy}{dx} = \lim_{h\to 0} \frac{e^x \left[1 + h + \frac{h^2}{2\,!} + \frac{h^3}{3\,!} + \ldots - 1\right]}{h}$$

$$\Rightarrow \quad \frac{dy}{dx} = \lim_{h\to 0} \frac{e^x \left[h + \frac{h^2}{2\,!} + \frac{h^3}{3\,!} + \ldots\right]}{h}$$

$$\Rightarrow \quad \frac{dy}{dx} = \lim_{h\to 0} e^x \left(1 + \frac{h}{2\,!} + \frac{h^2}{3\,!} + \ldots\right)$$

$$= e^x + 0 + 0 + \ldots$$

$$\therefore \quad \frac{d}{dx}(e^x) = e^x$$

14.2.5 Differential Coefficient of Logarithmic Functions by First Principle

(i) Differential Coefficient of $\log_e x$.

Let $\quad y = f(x) = \log_e x$

then $\quad f(x+h) = \log_e (x+h)$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{d}{dx}(\log_e x) = \lim_{h\to 0} \frac{\log_e (x+h) - \log_e (x)}{h}$$

$$= \lim_{h\to 0} \frac{1}{h} \log_e \left(\frac{x+h}{x}\right)$$

$$= \lim_{h\to 0} \frac{1}{h} \log_e \left(1 + \frac{h}{x}\right)$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{h}{x} - \frac{1}{2}\left(\frac{h}{x}\right)^2 + \frac{1}{3}\left(\frac{h}{x}\right)^3 \ldots\right]$$

$$= \left[\frac{1}{x} - 0 + 0 \ldots\right]$$

$$\therefore \quad \frac{d}{dx}(\log_e x) = \frac{1}{x}$$

(ii) Differential Coefficient of $\log_a x$.

Let
$$y = f(x) = \log_a x$$
$$f(x+h) = \log_a (x+h)$$
$$\frac{d}{dx} y = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$\frac{d}{dx}(\log_a x) = \lim_{h \to 0} \frac{1}{h} [\log_a (x+h) - \log_a x]$$
$$= \lim_{h \to 0} \frac{1}{h} \log_a \left(\frac{x+h}{x}\right)$$
$$= \lim_{h \to 0} \frac{1}{h} \cdot \frac{h}{x} \cdot \frac{x}{h} \log_a \left(1 + \frac{h}{x}\right)$$
$$= \lim_{h \to 0} \frac{1}{h} \cdot \frac{h}{x} \log_a \left(1 + \frac{h}{x}\right)^{x/h}$$
$$= \frac{1}{x} \lim_{h \to 0} \log_a \left(1 + \frac{h}{x}\right)^{x/h}$$
$$= \frac{1}{x} \log_a e \qquad \left[\because \lim_{x \to 0} (1+x)^{1/x} = e\right]$$
$$\frac{d}{dx}(\log_a x) = \frac{1}{x} \log_a e$$

ILLUSTRATIVE EXAMPLES

Example 14.1. *Differentiate $3x^4$ from first principle.*

Sol. Let $\quad f(x) = 3x^4$
$$f(x+h) = 3(x+h)^4$$
By definition,
$$\frac{d}{dx}(3x^4) = \lim_{h \to 0} \frac{3(x+h)^4 - 3x^4}{h}$$
$$= \lim_{h \to 0} \frac{3x^4 \left\{\left(1 + \frac{h}{x}\right)^4 - 1\right\}}{h}$$
$$= 3 \lim_{h \to 0} \frac{x^4 \left\{1 + 4\left(\frac{h}{x}\right) + \frac{4.3}{2!}\left(\frac{h}{x}\right)^2 + \ldots - 1\right\}}{h}$$
$$= 3 \lim_{h \to 0} \frac{x^4 \left[\frac{4h}{x} + \frac{6h^2}{x^2} + \ldots\right]}{h}$$
$$= 3 \lim_{h \to 0} \left[\frac{4x^4}{x} + \frac{6x^4 h}{x^2} + \ldots\right]$$
$$= 3 \lim_{h \to 0} \left[4x^3 + 6x^2 h + \ldots\right]$$
$$= 3(4x^3) + 0 + \ldots$$
$$\frac{d}{dx}(3x^4) = 12x^3 \qquad \textbf{Ans.}$$

Example 14.2. *Differentiate $\cos 4x^2$ by the first principle.*

Sol. Let $\quad f(x) = \cos 4x^2$

$\therefore \quad f(x+h) = \cos 4(x+h)^2$

By definition,
$$\frac{d}{dx}(\cos 4x^2) = \lim_{h \to 0} \frac{\cos 4(x+h)^2 - \cos 4x^2}{h}$$
$$= \lim_{h \to 0} \frac{2 \sin \frac{4(x+h)^2 + 4x^2}{2} \sin \frac{4x^2 - 4(x+h)^2}{2}}{h}$$
$$= \lim_{h \to 0} \frac{-2 \sin (4x^2 + 4xh + 2h^2) \sin (2h^2 + 4xh)}{h}$$
$$= \lim_{h \to 0} \frac{-2 \sin (4x^2 + 4xh + 2h^2)(2h + 4x) \sin (2h^2 + 4xh)}{h(2h + 4x)}$$
$$= \lim_{h \to 0} -2 \sin (4x^2 + 4xh + 2h^2)(2h + 4x) \lim_{h \to 0} \frac{\sin (2h^2 + 4xh)}{(2h^2 + 4xh)}$$
$$= -2 \sin (4x^2)(4x) \,.\, 1$$
$$\frac{d}{dx}(\cos 4x^2) = -8x \sin 4x^2 \qquad \textbf{Ans.}$$

Example 14.3. *Find the differential coefficient of $x \sin x$.*

Sol. $\quad f(x) = x \sin x$

$\therefore \quad f(x+h) = (x+h) \sin (x+h)$

CHAPTER 20

STATISTICS

20.1 INTRODUCTION

Statistics is an important branch of mathematics, its field is not limited upto mathematics. It is useful in other branches as well. Statistics may be defined as the science of collection, organization, presentation, analysis and interpretation of numerical data. Statistics means quantitative information or quantification of the facts and findings.

According to Bowely "Statistics is the science of the calculations and averages."

According to Louit "Statistics is the science in which the collection, classification and tabulation of the numerical facts are done for the explanation, information and comparison of any event."

20.2 MEASURES OF CENTRAL TENDENCY

The number which represents a particular group of facts is called the average of that fact. Average is also called statistical average or mean. As the average of any group of facts cannot be a maximum value or minimum value of the group hence, it should lie at about the centre of the groups. The special feature about a mean is that most of the members of that group are concentrated around the mean. Due to this reason only mean is called the measures of central tendency. Generally there are following three types of measures of central tendencies :

(i) arithmetic mean, (ii) mode,
(iii) median.

20.2.1 Arithmetic Average or Mean : Arithmetic mean of a group of observations is the quotient obtained by dividing the sum of all the observations by their number. Thus arithmetic mean denoted by $\overline{X}$ or a. Arithmetic mean are of following two types :

(i) simple arithmetic mean,
(ii) weighted arithmetic mean.

20.2.1.1 Merits of Arithmetic Mean : The main merits of arithmetic mean are as follows :

(i) It is affected by the presence of every item in the group.

(ii) It can be estimated even without arranging data in a set form.

(iii) It can be easily calculated and is simple to understand.

(iv) If the number of observations or items is large, it is very reliable basis for comparison.

(v) It is uniquely defined i.e., there is one and only one value of arithmetic mean for any certain group of data.

20.2.1.2 Demerits of Arithmetic Mean : The main demerits of arithmetic mean are as follows :

(i) If any term is missing, it cannot be calculated.

(ii) It may be too much affected by extreme and lowest values and may not be representative of the group.

(iii) It may give fallacious and misleading results which may not be true in actual life e.g., 8·5 students.

(iv) It cannot be used in finding ratio, rate etc.

(v) The mean for the attributes which have no numerical measurements as prettyness, honesty, cleverness etc. cannot be calculated.

20.2.1.3 Utility of Arithmetic Mean : Arithmetic mean is used to calculate average income, average cost, average production, average capital etc.

20.2.1.4 Calculation of Arithmetic Mean : The methods of calculation of arithmetic mean are as follows :

(i) Calculation of Arithmetic Mean in Individual Series : In the case of individual series, the arithmetic mean may be calculated by following two methods :

(a) Direct Method : When all the values of any variable are added up and the sum so obtained is divided by the total number of values, it is called the average value of that variable.

If n values of a variable are $x_1, x_2, x_3 \dots x_n$ then their arithmetic mean is

$$\overline{X} \text{ or } a = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$$

$$= \frac{\Sigma n}{n}$$

Example : *Pocket money of 10 students is Rs. 15, 20, 30, 22, 25, 18, 40, 50, 55 and 65. Find out average pocket money.*

Sol. Required arithmetic mean $= \frac{\Sigma x}{n}$

$$\overline{X} = \frac{15 + 20 + 30 + 22 + 25 + 18 + 40 + 50 + 55 + 65}{10}$$

$$= \frac{340}{10} = 34$$

(b) Short-cut Method : According to this method

$$\text{Arithmetic mean } \overline{X} \text{ or } a = A + \frac{\Sigma dx}{n}$$

where A = Assumed mean

$dx = x - A$

Example : *The monthly income of a 5 persons is as 132, 140, 144, 136 and 138. Calculate the arithmetic mean.*

Sol.

S.No.	Monthly income (in Rs) x	Deviation $dx = x - A$
1	132	− 12
2	140	− 4
3	144 = A	0
4	136	− 8
5	138	− 6
		$\Sigma dx = -30$

$$\therefore \quad \overline{X} = A + \frac{\Sigma dx}{n}$$

$$= 144 - \frac{30}{5} = 144 - 6$$

$$= \text{Rs } 138$$

(ii) Calculation of Arithmetic mean in Discontinuous or Discrete Series : In discontinuous or discrete series the arithmetic mean may be calculated by following two methods :

(a) Direct Method : If n values of a variable x are $x_1, x_2, x_3, \dots, x_n$ and $f_1, f_2, f_3, \dots, f_n$ be the corresponding frequencies, then

$$\text{Arithmetic mean} = \frac{f_1x_1 + f_2x_2 + f_3x_3 + \dots + f_nx_n}{f_1 + f_2 + f_3 + \dots + f_n}$$

$$= \frac{\Sigma fx}{\Sigma f} = \frac{\Sigma fx}{n}$$

where, f = Frequency

x = Term (given value)

Σf or n = Sum of total frequencies

Σfx = Term × Sum of total frequencies

Example : *Calculate the arithmetic mean from the following data :*

Variable (x)	:	5	10	15	20	25	30	35	40
Frequency (f)	:	7	8	9	11	6	11	3	2

Sol.

x	f	fx
5	7	35
10	8	80
15	9	135
20	11	220
25	6	150
30	11	330
35	3	105
40	2	80
	$\Sigma f = 57$	$\Sigma fx = 1135$

$\therefore$ Arithmetic mean, $\overline{X} = \dfrac{\Sigma fx}{\Sigma f}$

$$= \frac{1135}{57} = 19{\cdot}91$$

(b) *Short-cut Method :* According to this method

Arithmetic mean, $\overline{X} = A + \dfrac{\Sigma f\,dx}{n}$

where A = Assumed mean

$dx = x - A$ = Deviation from assumed mean

Example : *Find the arithmetic mean of the following series :*

Height in cm	:	65	66	67	68	69	70	71	72	73
No. of plants	:	1	4	5	7	11	10	6	4	2

Sol. Let the assumed mean $A = 69$

Height (in cm) x	No. of Plants f	dx = x – A	fdx
65	1	– 4	– 4
66	4	– 3	– 12
67	5	– 2	– 10
68	7	– 1	– 7
69	11	0	0
70	10	1	10
71	6	2	12
72	4	3	12
73	2	4	8
	$\Sigma f = n = 50$		$\Sigma f\,dx = 9$

$\therefore$ Arithmetic mean $= A + \dfrac{\Sigma f\,dx}{n}$

$$= 69 + \frac{9}{50} = 69 + 0{\cdot}18$$

$$= 69{\cdot}18$$

(iii) Calculation of Arithmetic Mean in Continuous Series : First of all we calculate the midvalues of all classes and then we suppose that the frequency of a class is concentrated at its midvalue. Then proceeding as in discontinuous series we find the arithmetic mean.

In continuous series the arithmetic mean may be calculated by following two methods :

(a) *Direct Method :* According to this method

Arithmetic mean, $\overline{X} = \dfrac{\Sigma fx}{\Sigma f}$.

Example : *Find the arithmetic mean of the following series :*

Marks	:	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
No. of Students	:	10	16	14	7	3

Sol.

Marks	Midvalue x	Frequency f	fx
0 – 10	5	10	50
10 – 20	15	16	240
20 – 30	25	14	350
30 – 40	35	7	245
40 – 50	45	3	135
		$\Sigma f = 50$	$\Sigma fx = 1020$

$\therefore$ $\overline{X} = \dfrac{\Sigma fx}{\Sigma f}$

$$= \frac{1020}{50}$$

$$= 20{\cdot}4$$

(b) *Short-cut Method :* According to this method

Arithmetic mean, $\overline{X} = A + \dfrac{\Sigma f dx}{\Sigma f}$

Example : *Find the arithmetic mean from the following data :*

Marks	:	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
No. of Students	:	7	9	12	8	4

Sol. Let the assumed mean $A = 25$

Marks	Midvalue x	Frequency f	$dx = x - A$	fdx
0 – 10	5	7	– 20	– 140
10 – 20	15	9	– 10	– 90
20 – 30	25	12	0	0
30 – 40	35	8	10	80
40 – 50	45	4	20	80
		$\Sigma f = 40$		$\Sigma f dx = -70$

$$\therefore \quad \overline{X} = A + \frac{\Sigma fdx}{\Sigma f}$$

$$= 25 - \frac{70}{40} = 25 - 1{\cdot}75$$

$$= 23{\cdot}25$$

(iv) Calculation of Arithmetic Mean by Step Deviation Method : Step deviation method is a very useful method of calculating arithmetic mean in case of class intervals are same.

$$\text{Arithmetic mean, } \overline{X} = A + \frac{\Sigma fu}{\Sigma f} \times i$$

where, i = Class interval

$$u = \frac{x - A}{i}$$

***Example :** Find the arithmetic mean from the following data :*

Class Interval	:	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25
Frequency	:	4	7	5	9	0

Sol. Let the assumed mean $A = 12{\cdot}5$ and $i = 5$

Class Interval	Midvalue x	Frequency f	Deviation $dx = x - A$	$u = \frac{x - A}{i}$	fu
0 – 5	2·5	4	– 10	– 2	– 8
5 – 10	7·5	7	– 5	– 1	– 7
10 – 15	12·5	5	0	0	0
15 – 20	17·5	9	5	1	9
20 – 25	22·5	0	10	2	0
		$\Sigma f = 25$			$\Sigma fu = -6$

$$\therefore \quad \overline{X} = A + \frac{\Sigma fu}{\Sigma f} \times i$$

$$= 12{\cdot}5 + \left(\frac{-6}{25}\right) \times 5$$

$$= 12{\cdot}5 - 1{\cdot}2$$

$$= 11{\cdot}3$$

(v) Calculation of Arithmetic Mean in a Inclusive Series : Calculation of arithmetic mean of inclusive series is the same as of exclusive series.

***Example :** Find the arithmetic mean from the following data :*

Class Interval	:	20 – 29	30 – 39	40 – 49	50 – 59	60 – 69
Frequency	:	10	8	6	4	2

Sol. Let the assumed mean $A = 44{\cdot}5$

Class Interval	Midvalue x	Frequency f	Deviation $dx = x - A$	fdx
20 – 29	24·5	10	– 20	– 200
30 – 39	34·5	8	– 10	– 80
40 – 49	44·5	6	0	0
50 – 59	54·5	4	10	40
60 – 69	64·5	2	20	40
		$\Sigma f = 30$		$\Sigma f dx = -200$

$$\therefore \quad \text{Arithmetic mean, } \overline{X} = A + \frac{\Sigma f dx}{\Sigma f}$$

$$= 44{\cdot}5 + \left(\frac{-200}{30}\right) = 44{\cdot}5 - 6{\cdot}67$$

$$= 37{\cdot}83$$

(vi) Calculation of Arithmetic Mean in Case of Cumulative Frequency Distribution : Converting cumulative frequency distribution into a simple frequency distribution.

***Example :** Find the arithmetic mean from the following data :*

Marks Obtained (More than)	:	50	100	150	200	250
No. of Students	:	80	76	55	40	32

Sol. Let the assumed mean $A = 125$

Marks Obtained (More Than)	Midvalue x	No. of Students f	$dx = x - A$	fdx
0 – 50	25	80 – 76 = 4	– 100	– 400
50 – 100	75	76 – 55 = 21	– 50	– 1050
100 – 150	125	55 – 40 = 15	0	0
150 – 200	175	40 – 32 = 8	50	400
200 – 250	225	32 – 0 = 32	100	3200
		80		2150

$$\therefore \quad \overline{X} = A + \frac{\Sigma f dx}{\Sigma f}$$

$$= 125 + \frac{2150}{80} = 125 + 26{\cdot}875$$

$= 151.875$

Example : *Find the arithmetic mean from the following data :*

Marks Obtained (Less Than)	:	10	20	30	40	50	60	70	80
No. of Students	:	5	13	20	32	60	80	90	100

Sol. Let the assumed mean $A = 45$

Marks Obtained	Midvalue x	No. of Students f	$dx = x - A$	$f\,dx$
0 – 10	5	5	– 40	– 200
10 – 20	15	13 – 5 = 8	– 30	– 240
20 – 30	25	20 – 13 = 7	– 20	– 140
30 – 40	35	32 – 20 = 12	– 10	– 120
40 – 50	45	60 – 32 = 28	0	0
50 – 60	55	80 – 60 = 20	10	200
60 – 70	65	90 – 80 = 10	20	200
70 – 80	75	100 – 90 = 10	30	300
		100		0

$\therefore$ Arithmetic mean $= A + \dfrac{\Sigma f\,dx}{n}$

$$= 45 + \frac{0}{100}$$

$$= 45 + 0$$

$$\overline{X} = 45$$

ILLUSTRATIVE EXAMPLES

Example 20.1. *The monthly income of 5 families are as follows :*

Family	:	*P*	*Q*	*R*	*S*	*T*
Monthly Income (in Rs)	:	744	750	753	758	761

Find the arithmetic mean by direct and short cut method.

Sol. Let the assumed mean $A = 753$

Family	Monthly Income (in Rs) x	Deviation $dx = x - A$
P	744	– 9
Q	750	– 3
R	753	0
S	758	5
T	761	8
	3766	1

(i) By direct method

Arithmetic mean, $\overline{X} = \dfrac{\Sigma x}{n}$

$$= \frac{3766}{5}$$

$$= \text{Rs } 753.2 \qquad \textbf{Ans.}$$

(ii) By short cut method

Arithmetic mean, $\overline{X} = A + \dfrac{\Sigma dx}{n}$

$$= 753 + \frac{1}{5} = 753 + 0.2$$

$$= \text{Rs } 753.2 \qquad \textbf{Ans.}$$

Example 20.2. **Find the arithmetic mean from the following data :**

Measure	:	6	8	10	12	14	16	18	20
Frequency	:	4	3	2	4	3	1	6	4

Sol.

Measure x	Frequency f	fx
6	4	24
8	3	24
10	2	20
12	4	48
14	3	42
16	1	16
18	6	108
20	4	80
	27	362

$\therefore$ Arithmetic mean $= \dfrac{\Sigma fx}{\Sigma f}$

$$= \frac{362}{27}$$

$$\overline{X} = 13.40 \qquad \textbf{Ans.}$$

Example 20.3. **Find the arithmetic mean from the following data :**

Marks Obtained	:	5 – 10	10 – 15	15 – 20	20 – 25
No. of Students	:	6	5	15	10
Marks Obtained	:	25 – 30	30 – 35	35 – 40	40 – 45
No. of Students	:	5	4	2	2

Sol. Let the assumed mean $A = 27.5$